

Time Dependent Failure Analysis of Compressed Riprap as Riverbank Protection

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ABSTRACT: Riprap is a common measure for protecting river banks against lateral erosion and several methods exist to design them. However, they are generally limited to dumped medium size rocks. If large rock blocks are required for stability reasons, they cannot be dumped anymore but they have to be placed individually. This gives additional resistance against flow erosion. The behavior of large rock blocks for alpine river embankments was so far rarely studied. Thus, an experimental investigation was conducted to investigate the stability of such compressed large blocks as river bank protection. Experiments were conducted in a 10 m long, 1 m wide flume with a rough fixed bed. Riprap was reproduced with uniform crushed stones having three different median block sizes of $D_{50}=0.037$, 0.042 and 0.047 m. Testing was undertaken for a stream-wise slope of 0.03 in supercritical flow conditions. Transversal slope of the riprap was 3.5V-5H. One layer of blocks as well as two layers was studied in order to investigate the influence of the riprap thickness. They were compressed and placed on the filter with a wide grain size distribution. The riprap failure threshold discharge was determined based on series of tests with duration of 3 hours. Riprap erosion rate was measured with a one minute frequency, by means of block tracking with three cameras and standard video-image processing techniques. Furthermore, the eroded rocks were collected and weighed in a sediment trap at the downstream end of the channel. The time of first total failure was recorded and a time based analysis of failure was performed. Total collapse of the blocks in a reach over the whole bank was defined as full failure. First results revealed that for similar unit discharge the rate of block erosion is significantly decreased by increasing the size of the riprap. The time of failure depends also on the size of blocks. It could be observed that under the same conditions, the upper layer stabilized the protection system and delayed the time of total failure. Nevertheless, erosion rate of the upper layer of rock blocks increased.

KEY WORDS: Bank Erosion, Riprap, Failure, Sliding, Time dependent analysis.

1 INTRODUCTION

While many different methods have been used to protect river banks, riprap is still the most commonly used protection in gravel bed rivers. Riprap is long lasting, flexible, easily constructed, and natural in appearance if correctly placed (Schleiss, 1998). Considerable effort has been allocated to develop methods to design riprap. In these methods riprap block size, gradation, thickness (number of layers) and characteristics of filter behind it has been defined as important design parameters (Stevens et al., 1976; Maynard et al., 1987; Escarameia and May, 1995).

Riprap failure occurs under different mechanisms. According to Julien (2002) and Lagasse et al.

(2006), riprap failure modes are identified as direct block erosion, translational slide, slump, and side-slope failure. Direct block erosion initiated by flowing water is the most common mechanism of erosion. Inadequate size of blocks is one of the reasons for direct erosion as well as steep slope and too uniform gradation of riprap. Translational slide is a failure occurred by the downslope riprap material movement. The initial phases of this collapse are showed by cracks in the upper part of the riprap blanket that extend parallel to the channel. This failure process mostly occurs due to toe scouring and instability of the riprap caused by the weakness in the toe foundation. It might be also the consequence of direct block erosion. Modified slump failure of riprap is similar to translational slide except this occurs as the slide of different layers of stones on each other. Probable causes of modified slump are steep slope of embankment and lack of toe support. Side-slope failure of the riprap is caused mostly due to overtopping and it corresponds to a rotation-gravitational movement of riprap. While overtopping occurs, the water saturates the riprap and the material behind it. Once the level of the water decreases, the water in the saturated part tend to steep pressure gradient and the slide-slope in riverbank riprap takes place (Jafarnejad et al., 2011).

There are different methods available to design riverbank ripraps resisting against direct block erosion. Some of them are reviewed and discussed in detail by Maynard and Neil (Garcia, 2007). Several manuals, such as Centre for Civil Engineering Research and Code (CUR, 1995) presented dumped block riprap design and their general applications. In this Manual an equation developed by Pilarczyk (1990) for stability of the riprap is presented concerning strengthening and destabilizing forces. Stevens et al. (1976) presented a safety factor based method by considering the stability of individual blocks in riprap. They assumed that each block is stable if the several forces causing a possible displacement of a block represent less than the reaction caused by the submerged weight. Wittler and Abt (1988) modified Stevens' study adding frictional and contact forces from adjoining blocks. Froehlich and Benson (1996) also worked on wide angle of repose to refer the slope of embankment effect on riprap stability. They proposed a "particle angle of initial yield" which was as well introduced earlier by Straub (1953), Grace et al. (1973), and Reese (1984).

U.S. Corps of Engineers Manual (USACE, 1994) presented a method for sizing riprap in rivers and channels based on different coefficients concerning incipient failure, vertical velocity distribution, and thickness. This method was mainly based on Maynard's formula presented in 1987. Brown and Clyde (1989) used both the Manning-Strickler equation with the Shields relation to make a combined formula for the size of stable rocks. Escaramia and May (1992) presented a general equation for design riverbank ripraps and gabion mattresses. Stability of loose rock riprap was also studied by Froehlich (2011) regarding the protection of stream banks from erosive forces and evaluation based on the ratio of static moments resisting overturning. The ratio of moments in this research defined a safety factor which indicates the potential of riprap failure. Abt et al. (2008) studied the round-shaped riprap stabilization in overtopping flow as well. Froehlich (2011), Ulrich (1987) and Stevens et al. (1984) also considered the weight of the submerged rock as the only resisting force. Probabilistic procedures for design of riverbank riprap were developed by Li et al. (1976), PIANC (1987), and later by Froehlich and Benson (1996). Combination of different mechanisms and persistence on risk-based design procedure is one of the advantages of these kinds of methods to apply more in future.

Existing design methods are generally limited to dumped medium size rocks and incipient motion of particles is used as failure criterion (De Almeida and Martin-Vide, 2009). However, if large heavy blocks are required for stability reasons, they have to be placed individually because of their weight. According to Schleiss (1998) the critical shear stress of these large mountain blocks are $\theta_{cr}=0.1$ instead of Shields critical shear stress as 0.047. Therefore, in large compressed boulders as rock ripraps, erosion of one block is not the reason of total failure due to the extra support of compressed stones. Failure happens when a group of blocks slides and makes the river bank unstable. In the present study, the first observation of total collapse of the blocks along the river bank slope (as a slide or slump) is defined as failure criterion.

Accordingly, this paper reports about the influence of the size and thickness of large blocks individually positioned as riprap. This is succeeded through flume experiments investigating the behavior of compressed large blocks and their entrainment by the flow. The present article is based on the result of eighteen series of experiments conducted to evaluate the failure process. The analysis focused mainly on

characteristic time of failure and critical hydraulic parameters.

2 EXPERIMENTAL PROCEDURE

The main goal of this research was evaluating the resistance of river bank riprap protection constituted by individually placed, compressed large blocks, subjected to hydrodynamic forces. Eighteen systematic experiments were conducted to analyze the impact of block size and thickness on the stability of compressed riprap. These laboratory tests were carried out in a straight 10 m long, 1 m wide and 0.5 m deep tilting flume with a trapezoidal section of a laboratory of hydraulic constructions at Ecole Polytechnique Fédérale de Lausanne. The channel was fed by water from the internal closed circuit by means of a pumping system that allowed maximum discharge of 750 l/s. A schematic sketch of the longitudinal and cross sections of the setup is shown in Figure 1.

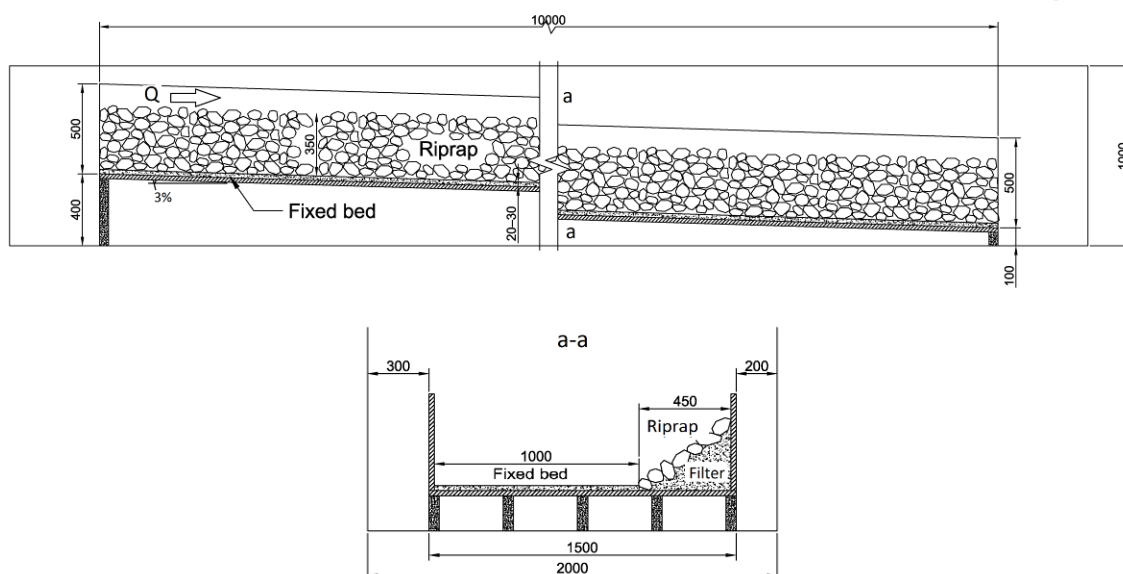


Figure 1 Sketch of longitudinal and cross-section view of the experimental flume (units in mm)

The bottom slope of the flume was fixed to 3% and the transversal riprap slope of the riprap as 3.5V-5H (35°). Studied riprap material included uniform crushed stones with three different median rock sizes of $d_B=0.37$, 0.42 and 0.47 m. Blocks were compressed and placed over a wide grain size distribution filter. In order to simulate the natural hydraulic conditions, the roughness of the natural river bed was reproduced with the same material of the filter, glued on the bed of the channel (Table 1).

Table 1 Grain size distribution of the filter and the river bed

D_m (mm)	D_{10} (mm)	D_{35} (mm)	D_{50} (mm)	D_{75} (mm)	D_{90} (mm)	D_{max} (mm)
8.5	3.2	4.4	5.3	9.1	14.8	32

The experimental program was set up to identify which the critical unit discharge caused the failure and when this failure occurred. Lower discharges may cause direct block erosion during the experiments but not full failure of the bank slope. Tests were carried out under supercritical flow conditions and with a constant discharge to reach the threshold unit discharge (Table 2). Discharge, water depth, block erosion rate and time of failure were measured during the tests. Water discharge supplied by two pumps, was measured by electromagnetic flow meters with of ± 2 l/s of precision. Water depth was measured by ultrasonic distance measuring sensors with a precision of ± 0.5 mm. To avoid the influence of the model inlet, the first 1 m of the riprap was fixed at the upstream by gluing the rocks together with cement.

Riprap erosion rate was measured with a one minute frequency by tracking and counting the eroded blocks with three cameras. Furthermore, the eroded rocks were collected and weighed in a sediment trap at the downstream end of the channel. To reduce the construction effects and to avoid perturbations from the initial arrangement, blocks eroded before the discharge stabilization (less than 2 minutes) were not considered. Moreover, the results are based on the part of flume where water depth remained roughly constant (7 m downstream from inlet). The eroded blocks from upstream of that zone were thus excluded as well. During the tests, the water depth was measured using ultrasonic sensors in four different positions with 2 m distance in the test reach. They were all transversally located at the center of the channel cross section.

The detailed characteristics of the experiments are presented in Table 2. Experiments were divided in four groups. Group I, II and III, corresponding to three different block sizes (0.037, 0.042 and 0.047 m, respectively) were tested with one layer riprap installation whereas group IV included two layers of same size blocks ($d_B = 0.037$ m). Each run of the tests compounds to a specific constant discharge. Tests were run during three hours unless the total failure of the blocks occurred in a section. Therefore, riprap failure threshold discharge was determined based on series of tests with duration of maximum 180 minutes. Figure 2 shows an example the view before and after one test (the test IV-1 in this case, cf. Table 2). This particular test was carried out for two layers of stones. Figure 2-(a) presents the set up before the test, where only the external layer (white stones) can be seen. Eroded parts of riprap in both layers and the failed area after the experiment can be observed in Figure 2-(b).

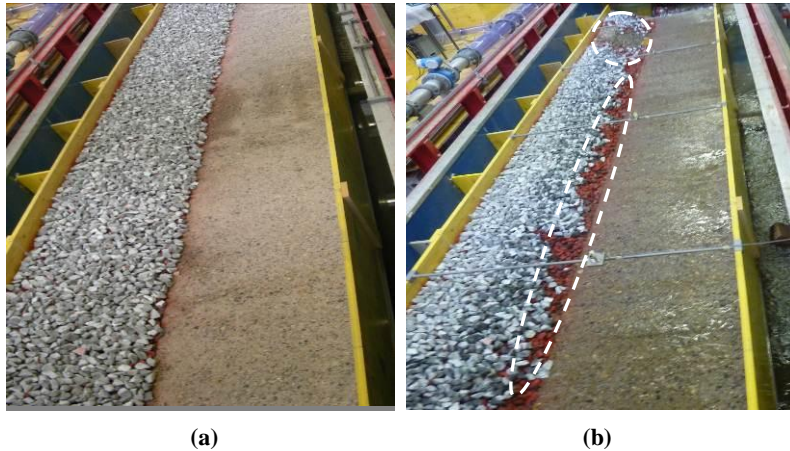


Figure 2 The experimental set up before (a) and after (b) one test (the test IV-1, cf. Table 2)

The temporal evolution of the failure of the riprap protection with well positioned blocks is investigated herein. One of the issues which not yet fully known in the design of riprap protection is the influence of time (flow duration) on their stability. One approach to consider time is to treat riprap design as a transport problem defining maximum allowable transport rates. This approach is acceptable when multiple layers of material are considered (Maynard et al., 1987). However, it may be questionable when one or two layers of large blocks are used. Another approach is determining how the various hydrographs over a given project form a total time to use in design (life time of the project).

Typical prototype time corresponding to the maximum 180 minutes duration is an important consideration for the application. The prototype to model time ratio based on Froude similarity is:

$$\lambda_\tau = \frac{T_p}{T_m} = \sqrt{\frac{L_p}{L_m}} \quad (1)$$

Where T is time scale, L is length scale and subscripts p and m stand for prototype and model values, respectively. For assuming $L_p = 25 \times L_m$, considering a geometry scale of the experimental set-up for typical Swiss mountain rivers, the time ratio is $\lambda_\tau = 5$. Experimental tests lasting for three hours are thus

roughly equivalent to prototype floods with durations of 15 hours which is corresponded to the typical duration of a flood.

3 RESULTS AND DISCUSSION

In group I to III of the experiments, one layer of blocks was tested and water depth, rate of erosion and time of failure were measured to define the hydrodynamic effects on the erosion of compressed riprap. Two layers of same size blocks were also studied in order to investigate the influence of the riprap thickness (group IV). Table 2 gives the experimental program of the 18 tests with the parameters including unit discharges (q), water depth (h), size of the blocks (d_B), time to total failure (t_f), riprap condition after the end of the test (failed, F , or remained stable, S), mean velocity (v_m based on continuity), Froude number (Fr) and bed shear stress (τ) considering uniform flow conditions.

Table 2 Tests program

group	series	q (m ² /s)	h (m)	d_B (m)	t_f (min)	F or S (-)	v_m (m)	Fr (-)	τ (Pa)
I	I-1	0.301	0.165	0.037	62	F	2.13	1.67	48.56
	I-2	0.262	0.15	0.037	94	F	2.03	1.67	44.15
	I-3	0.249	0.145	0.037	-	S	1.99	1.67	42.67
	I-4	0.242	0.143	0.037	-	S	1.97	1.66	42.08
	I-5	0.208	0.131	0.037	-	S	1.84	1.62	38.55
	I-6	0.166	0.121	0.037	-	S	1.59	1.46	35.61
II	II-1	0.473	0.2	0.042	6	F	2.61	1.87	58.86
	II-2	0.442	0.193	0.042	14	F	2.52	1.83	56.80
	II-3	0.430	0.188	0.042	68	F	2.52	1.85	55.33
	II-4	0.421	0.186	0.042	121	F	2.49	1.84	54.74
	II-5	0.407	0.183	0.042	162	F	2.44	1.83	53.86
	II-6	0.380	0.175	0.042	-	S	2.38	1.81	51.50
	II-7	0.348	0.167	0.042	-	S	2.27	1.78	49.15
III	III-1	0.480	0.21	0.047	160	F	2.65	1.89	58.86
	III-2	0.461	0.196	0.047	-	S	2.59	1.87	57.68
	III-3	0.443	0.191	0.047	-	S	2.55	1.87	56.21
	III-4	0.432	0.188	0.047	-	S	2.53	1.85	55.92
IV	IV-1	0.299	0.163	0.037	87	F	2.13	1.69	47.97
q , h , d_B , t_f , v_m , Fr and τ : represent unit discharge, water depth, diameter of blocks, time of total failure, mean velocity, Froude Number and bed shear stress; F : failed; S : remained Stable									

Figure 3 shows two examples of the riprap before (a) and after (b) experiments I-2 and II-4. In test number I-2 with the smallest size of blocks (red stones) and a unit discharge of $q=0.262$ m²/s, direct block erosion occurred at test start. However, the total failure was observed 94 minutes afterwards and it occurred 8 m from the upstream section of the channel. The same process happened for test II-4 with medium size of blocks (yellow stones) and unit discharge of $q=0.421$ m²/s. The total failure happened 121 minutes after starting the experiment at almost the same section as in test I-2. It can be observed that in the failed sections the filter was fully exposed, whereas other areas of the bank protection were still covered and stable.

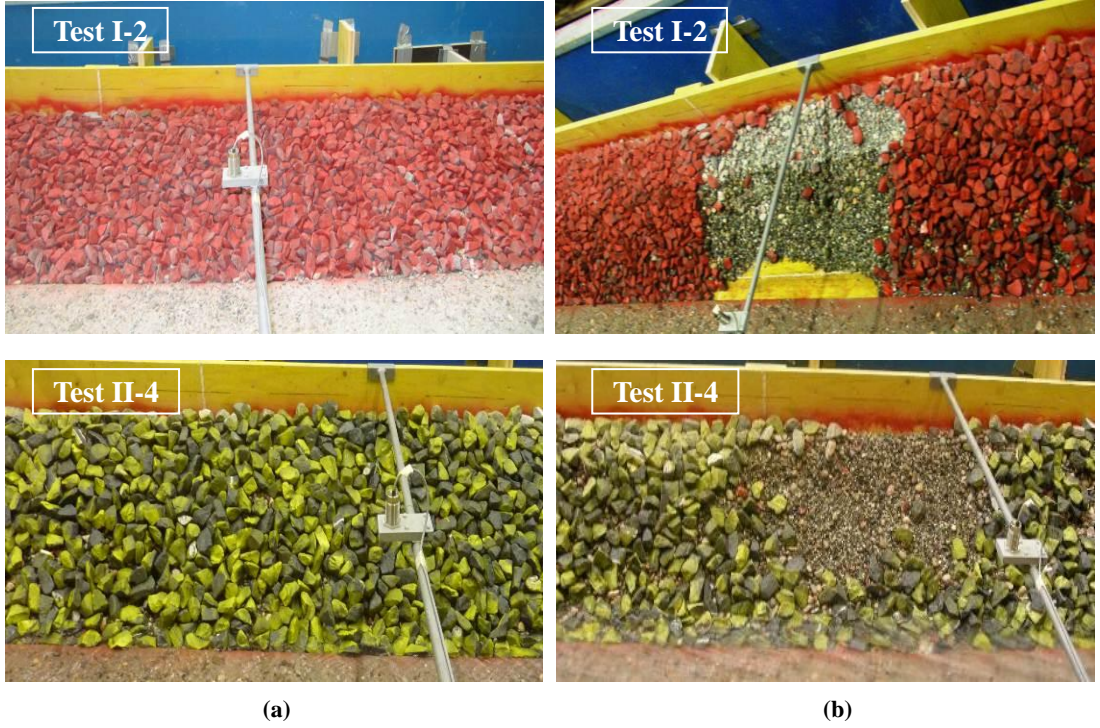


Figure 3 Riprap before (a) and after (b) failure for tests I-2 and II-4 (see table 2).

Figure 4 shows the time evolution of the cumulated number of eroded blocks. Time of failure (t_f) is normalized by the duration of the tests $T_{\max}=180$:

$$t^* = \frac{t_f}{T_{\max}} \quad (2)$$

Failure corresponding to the abrupt change of the slope of the graphic in Figure 4 is related to the transport of the material causing in the total failure of a bank protection.

Figure 4-(a) shows the series of tests in group I which included six different discharges. It can be observed that gradually increasing the constant discharge of the test I-1 to I-6 has an impact on the time of failure. Two of the tests in this group reached total failure and the critical unit discharge for this specific size compared to a value between $q=0.249 \text{ m}^2/\text{s}$ and $q=0.262 \text{ m}^2/\text{s}$ and with a failure time of roughly 1 hour and 30 minutes after beginning of the test. An increase of 14.9% of the unit discharge (to $q=0.301 \text{ m}^2/\text{s}$), resulted in an earlier failure (-17%).

In Figure 4-(b) shows the data of 7 different tests for medium size blocks. The significant impact of increasing discharge on the time of total failure and amount of eroded blocks can be clearly seen. Herein, the first total failure occurs for $q=0.407 \text{ m}^2/\text{s}$ at 90% of maximum time of the experiments (1 hour and 42 minutes). Thus by increasing the unit discharge of 3.3%, 5.6%, 8.6%, and 16.2% the time of failure was reduced, 23%, 57%, 82%, and 87% respectively.

For largest blocks ($d_B=0.047 \text{ m}$), failure condition was reached only for the test III-1 ($q=0.480 \text{ m}^2/\text{s}$). Figure 4-(c) indicates that the largest size of blocks (0.047 m) delayed the failure for a higher discharge by 15.2% as compared to the medium size and by 45.4% compared to the small sized blocks. For similar unit discharge the rate of block erosion is decreased by increasing the size of the riprap. Figure 4-(d) gives the comparison between one layer and two layers erosion rate for the same unit discharge. The second layer (lower layer) postponed the failure of the section significantly, namely around 50% of the total experiment duration. However the rate of erosion of upper layer is higher.

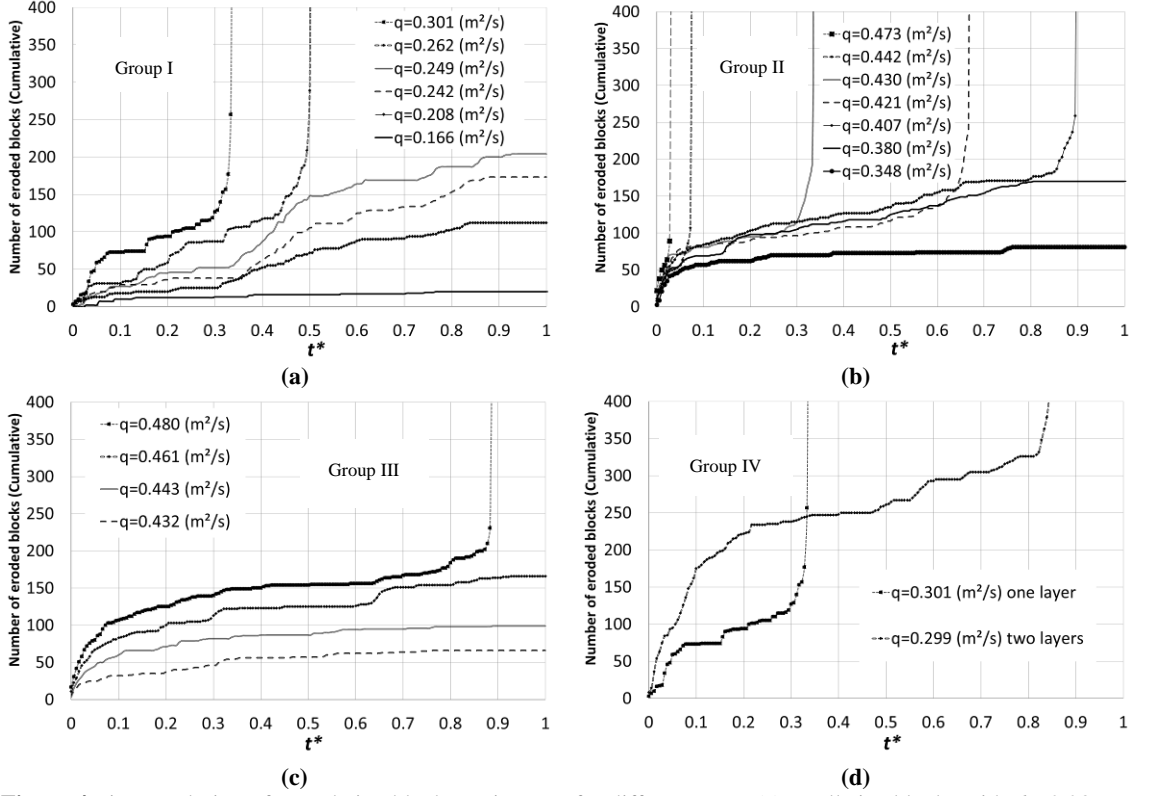


Figure 4 Time evolution of cumulative block erosion rate for different tests: (a) small size blocks with $d_B=0.037$ m; (b) medium size blocks with $d_B=0.042$ m; and large size blocks with $d_B=0.047$ m; (d) comparison of block erosion between one layer and two layers of riprap in size of $d_B=0.037$ m

In Figure 5 the relationship between dimensionless failure time t^* and (a) velocity, (b) Froude number, (c) bed shear stress and (d) dimensionless bed shear stress or Shields factor of one layer tests (group tests of I to III) is presented.

Figures 5-(a) to 5-(c) show the similarity and the results for the different block diameters apparently are in a same pattern. A plateau at $t^*=1$, corresponding to tests with no failure, is interpreted by a sudden decrease in the t^* values, corresponding to tests with failure. Results are grouped by block size. Nevertheless, for each block size, clear limit values above which failure of the riprap protection is expected exist for mean velocity, Froude number and bed shear stress,.

In terms of velocity, failure of the riprap is expected for small, medium and large sizes blocks for values above roughly 2.0, 2.4 and 2.6 m/s, respectively. For Froude number and bed shear stress, threshold limit limits are roughly lower than 1.68 and 43 Pa for small d_B , around 1.81 and 53 Pa for medium d_B and more than 1.88 and 58 Pa for large one. Curves corresponded to t^* should converge asymptotically to $t^*=0$. When any of these parameters increases, the fact can be inferred from the shape on the lower part of the curves for medium size block results.

In Figure 5-(a) to 5-(c) the failure time and consequently the occurrence of failure as a function of inertial flow resistance forces and block diameter is shown. Normalization of bed shear stress shows the similarity of the results in terms of failure time as seen in Figure 5-(d). This dimensionless bed shear stress also represents the balance of hydrodynamic forces acting on the riprap and the submerged weight of the blocks. Dimensionless shear stress is calculated as follows:

$$\tau^* = \frac{\tau}{(S_B - 1)g\rho d_B} \quad (3)$$

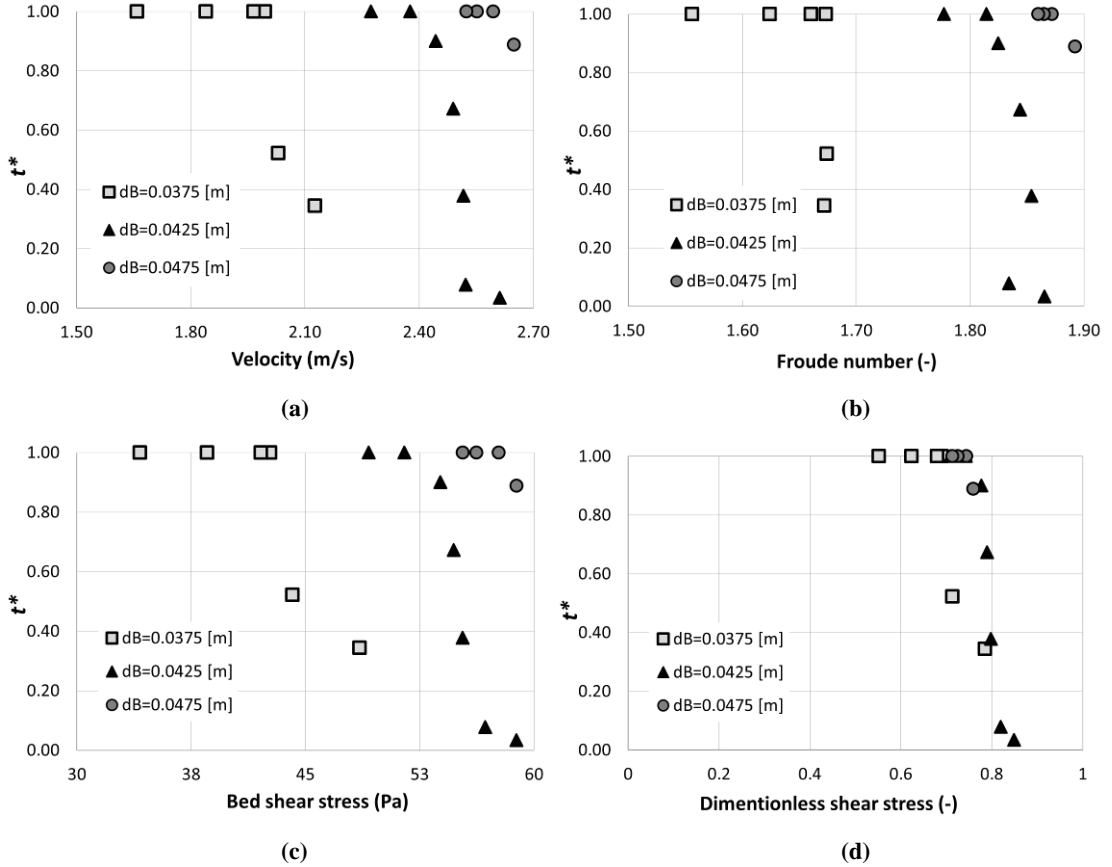


Figure 5 Normalized failure time versus mean velocity (a), Froude number (b), bed shear stress (c), and dimensionless shear stress (d)

where τ is the shear stress, S_B is the specific gravity of rocks, g is the gravitational acceleration, ρ is water density and d_B is the size of blocks. Figure 5 represents that the results of all groups of one layer tests that have the same trend regardless the size of rock blocks.

It can be also witnessed that the normalized time as $t^*=1$ characterizes the equilibrium which means that the riprap can stay stable regardless the time changes (considering the scaled model).

4 CONCLUSIONS

The behaviour of compressed, well positioned riverbank protection riprap was analysed in this research, considering the influence of size of blocks and thickness of riprap layer. Time dependent analysis of failure was performed. The maximum three hours duration of the flume tests can cover roughly 15 hours of an extreme event by taking into account a typical geometry scale factor between the experiments and actual Swiss mountain rivers.

A remarkable relationship between the size of riprap and the time of failure was observed. Furthermore, not only the larger block sizes postponed the time of failure but also reduced the block erosion rate significantly. By considering the two layer test, first results revealed that under the same conditions, the second layer stabilizes the protection system significantly and delays the time of total failure. Nevertheless, the erosion rate of the upper layer of rock blocks increased.

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